American University of Sharjah

Department of Mathematics and Statistics

MTH 203 , Sample Questions for Midterm I, 2009-2012

MULTIPLE CHOICE/SHORT ANSWER SECTION

(Circle the correct answers. No justification is required.)

1. (3 pts) Which of the following lines is perpendicular to the plane $2x - 4y + z = 7$?
   (a) $\mathbf{r}(t) = \langle 2, -4, 1\rangle + t\langle -2, 1, 0\rangle$
   (b) $\mathbf{r}(t) = \langle -2, 1, 0\rangle + t\langle 2, -3, 7\rangle$
   (c) $\mathbf{r}(t) = \langle 1, 15, 3\rangle + t\langle -4, 8, -2\rangle$
   (d) $\mathbf{r}(t) = \langle 0, 0, 1\rangle + t\langle 2, 4, 7\rangle$
   (e) None of the above

2. (3 pts) Let $\mathbf{u}$ and $\mathbf{v}$ be non-parallel vectors. If $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$ then
   (a) $\mathbf{w} = \mathbf{0}$
   (b) $\mathbf{w} = \mathbf{v}$ is the only solution
   (c) $\mathbf{w}$ is orthogonal to v.
   (d) $\mathbf{w}$ is parallel to u
   (e) None of the above.

3. (3 pts) Let $f(x, y)$ be a function with partial derivatives

$$f_x(x, y) = \cos(y) \quad \text{and} \quad f_y(x, y) = -x\sin(y)$$

   Let $g(t) = f(2t, e^t - 1)$. Then $g'(0)$ equals
   (a) 0
   (b) 1
   (c) 2
   (d) e
   (e) none of the above.

WRITEN SECTION

(You must show your work and justify your answers.)

(a) (4pts) Find the area of the triangle with vertices $(0, 0, 1)$, $(0, 3, 0)$ and $(1, 1, 1)$.

(b) (5pts) Determine if the following limit exists. If the limit exists then find its value.

$$\lim_{(x,y)\to(0,0)} \frac{x^3 y}{x^6 + y^2}$$

Q1.(14%) a) Find the equation of the line through $(1, -2, 3)$ and perpendicular to the plane $2x + 3y - z = 5$

b) Find the distance between the point and the plane in part(a)

Q2 (10%) Find the equation of the plane through $P(2,1,-3)$ that contains the line $L: x=t,\ y=3-2t,\ z=1-t$.

Q3.(10%) Find the parametric equations for the line of intersection of the planes $2x+y-9z=5$ and $x-2y+13z=0$.

Q4. (6%)Let $f(x,y)=\sqrt{5+x+2y^2}$ a) Find and sketch the domain of f. b)Sketch the level curve of the surface that passes through the point $(2,1)$.

Q5. (14%) Let $f(x,y)=\left\{\begin{array}{cc} \dfrac{y^3-x^2y+4x^2}{x^2+3y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{array}\right\}.$

a) Is $f(x,y)$ continuous at $(0,0)$? Justify your answer?

b) Does $f_x(0,0)$ and $f_y(0,0)$ exist? Justify your answer?

Q6.(6%) Show that the indicated limit exists

$$\lim_{(x,y)\to(0,0)} \frac{x^2\ln(2x^2+1)}{x^2+2y^2}$$

Q7.(16%) a)If $z=f(x,y)$ is implicitly defined by $z\sin(xy)-2xy^2z=1+x^2+3z^2$, find $\dfrac{\partial z}{\partial x}$?

b) Use the chain rule to find $\dfrac{\partial w}{\partial y}$ if $w=rt+v^2r+ue^v$ with $t=xy^2z$, $r=x^2yz$, $u=xyz^2$, $v=e^{xyz}$.

Q8. (16%) Consider the paraboloid $f(x,y)=4x^2+9y^2$. a) Find the directional derivative of f at the point $(1/4,-1/6)$ in the direction from $(2,3)$ to $(3,1)$.

b) What is the maximum and minimum rate of change of f at the point $(0,-1)$.

c) Find the points on the paraboloid in part (a) at which the normal line is parallel to the line through $P(-2,4,3)$ and $Q(5,-1,2)$

Q9. (12%)a) Let $f(x,y)=\sqrt{20-x^2-7y^2}$. Find the linearization of f at $(2,1)$.

b) Use part (a) to approximate $f(1.95,1.08)$?

1. (5 Points) Find an equation of the sphere with a diameter whose endpoints are $P(-2,3,4)$ and $Q(4,-3,-2)$.

2. (5 Points) Find parametric equations of the line passing through the point $(2,3,4)$ and perpendicular to the plane $3x+2y-z=6$.

3. (5 Points) For the two lines whose parametric equations are given below, find their point of intersection.

$$\begin{array}{ll} x = -2 + t & x = 3 - 4s \\ y = 1 + 2t & y = 2 + s \\ z = 4 - 2t & z = -4 + 6s \end{array}$$

4. (10 Points) A plane contains the point $Q(2, 2, 1)$ and the line $x = 2t$, $y = 4-t$, $z = t$. Find its equation. Do the following easy steps

   (a) Find a point P that is on the line. Is this point also on the plane?

   (b) Find a direction vector for the line.

   (c) Find the vector $\overrightarrow{PQ}$. Is this vector on the plane?

   (d) Find a normal vector to the plane.

   (e) Now find the equation of the plane.

5. (10 Points) Find the distance between the parallel planes

$$\begin{aligned} x + y - z &= 0 \\ x + y - z &= -2 \end{aligned}$$

6. (10 Points) Given the line $\dfrac{x-1}{4} = \dfrac{y}{2} = \dfrac{z-3}{6}$ and the plane $2x + 3y = -5$.

   (a) Find the point at which they intersect.

   (b) Had the equation of the plane been $2x - 4y = -5$, then the line and the plane would not intersect. Give algebraic justification and interpret geometrically.

7. (15 Points)

   (a) Let $f(x, y) = 36 - 4x^2 - 9y^2$. Find and describe the level curve for f when $c = 11$.

   (b) Discuss $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x - y^2}{2x^2 + y}$.

   (c) Let $z = f(x, y) = \ln(x^2 - xy)$ and find f_x and f_{xy}.

8. (5 Points) The radius of a cylinder is increasing at the rate of 2 meters/sec and the height is increasing at the rate of 3 meters/sec. How fast is the volume of the cylinder increasing when the radius is 8 meters and the height is 11 meters? [This is a chain rule application. $V = \pi r^2 h$.]

9. (5 Points) For $f(x, y) = y\cos(x - y)$, find the maximum rate of change of f at the point $(\pi/, \pi)$ and the direction in which it occurs.

10. (10 Points) Consider the sphere $f(x, y, z) = x^2 + y^2 + z^2 - 9$, find

    (a) an equation of the plane tangent to the sphere at the point $(2, 2, 1)$

    (b) a parameteric equations for the line normal to the surface at $(2, 2, 1)$.

11. (10 Points) Let $f(x, y) = x^3 - 3xy + y^2$ and find all local max, min, and saddle point(s), if any.